THE SCALAR COEFFICIENT FORM EQUATION

A single dependent variable u is an unknown function on the computational domain. COMSOL Multiphysics determines it by solving the PDE problem that you specify. In coefficient form, the PDE problem reads

$$\begin{cases} e_a \dfrac{\partial^2 u}{\partial t^2} + d_a \dfrac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f & \text{in } \Omega \\ \mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g - h^T \mu & \text{on } \partial\Omega \\ hu = r & \text{on } \partial\Omega \end{cases} \quad (9\text{-}1)$$

where

- Ω is the computational domain—the union of all subdomains
- $\partial\Omega$ is the domain boundary
- **n** is the outward unit normal vector on $\partial\Omega$

The first equation in the list above is the PDE, which must be satisfied in Ω. The second and third equations are the boundary conditions, which must hold on $\partial\Omega$. The second equation is a *generalized Neumann* boundary condition, whereas the third equation is a *Dirichlet* boundary condition. This nomenclature and the second equation above deviate slightly from traditional usage in potential theory where a Neumann condition usually refers to the case $q = 0$. The generalized Neumann condition is also called a *mixed boundary condition* or a *Robin boundary condition*. In finite element terminology, Neumann boundary conditions are called *natural boundary conditions* because they do not occur explicitly in the weak form of the PDE problem. Dirichlet conditions are called *essential boundary conditions* because they restrict the trial space. Dirichlet boundary conditions often represent *constraints*.

This manual uses the following conventions:

- The symbol — is the vector differential operator (gradient), defined as

$$\nabla = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}\right)$$

  The space coordinates are denoted $x_1, \ldots, x_n$, where n represents the number of space dimensions.

- The symbol Δ stands for the Laplace operator

$$\frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_n^2}$$

- — □ (c —u) means

$$\frac{\partial}{\partial x_1}\left(c\frac{\partial u}{\partial x_1}\right) + \ldots + \frac{\partial}{\partial x_n}\left(c\frac{\partial u}{\partial x_n}\right)$$

- β □—u means

$$\beta_1\frac{\partial u}{\partial x_1} + \ldots + \beta_n\frac{\partial u}{\partial x_n}$$

where β_1 , ... , β_n are the components of the vector β.

Within COMSOL Multiphysics, you specify the coefficients c, α, γ, β, a, q, and h, and the terms f, g, and r. They can all be functions of the spatial coordinates.

- A PDE is *linear* when the coefficients depend only on the spatial coordinates (or are constants).
- A PDE is *nonlinear* if the coefficients depend on u or its derivatives (the components of —u).
- All the coefficients in the above equation are scalars except α, β, and γ, which are vectors with n components. The coefficient c can alternatively be an $n \times n$ matrix to model anisotropic materials. For more information see "Modeling Anisotropic Material" on page 205 in the *COMSOL Multiphysics User's Guide.*

The e_a coefficient in Equation 9-1 is a scalar or a matrix for time-dependent systems called the *mass* *matrix* (or mass coefficient). The d_a coefficient represents a damping term (however, if $e_a = 0$, then d_a is often called the mass coefficient). See "Solving Time-Dependent Problems" on page 247 for more information on time-dependent problems.